Resonant Properties of Nonreciprocal Ring Circuits*

F. J. TISCHER†

Summary—**The ring circuit investigated consists of a resonant ring guide coupled to a main guide. The properties can be described by the equations for the waves in the ring guide resulting from excitation in the main guide. The influence of nonreciprocity on the properties is investigated under conditions of varying coupling. The representation of the ring waves by the poles and zeros is chosen to permit interpretation of the results under the large variety of operational conditions with respect to coupling and nonreciprocity. The application for measuring the material constants of ferrites is discussed.**

INTRODUCTION

THE properties of ring circuits under different coupling conditions were described in a previous paper.[1] Applications and special cases have been considered in the literature.[2–5] The general properties of nonreciprocal ring circuits are the subject of this investigation.

The properties of nonreciprocal ring circuits are interesting from the viewpoint of different applications, one of which is the possibility to measure the properties of ferrite materials in a directionally coupled resonant ring guide.[6] In properly conducted experiments, the intrinsic tensor permeability can be determined.

Nonreciprocal conditions in ring circuits can be obtained by filling a part of the cross section of the ring guide along a section or along the total circumference with premagnetized ferrite material. As a consequence, the propagation constants have different values for waves progressing in the ring guide in both directions. Two cases are of primary significance. One occurs when the phase velocity is different in the two directions. The other case results from excessive attenuation in one direction.

Since the properties of ring circuits depend on the coupling conditions, it is convenient to study the waves in the ring guide under interaction with a waveguide to which it is coupled and from which the energy is fed into the ring. One can distinguish between different coupling conditions as directional, nondirectional, and semidirectional coupling. A fourth significant condition occurs if an additional discontinuity is introduced into the ring guide. This additional discontinuity takes into account the case where energy is coupled out of the ring guide into a second waveguide by a second coupling element.

The great number of significant operational conditions complicates the description of the properties. This description should show primarily the influence of the nonreciprocity on the resonant frequency or resonant frequencies and on the Q value. A valuable tool for describing the properties was found in the representation of the ring-wave amplitudes as rational functions by their poles and zeros. The method is derived in Appendix II for the case of the reciprocal ring guide. The interpretation of the results is further simplified if the coupling conditions are described by the wave-coupling factors. The derivations of the corresponding relations are shown in Appendix I.

THE SYSTEM UNDER INVESTIGATION

The system under investigation consists of a waveguide to which a ring guide is coupled as shown schematically in Fig. 1. The coupling element, shown in Fig. 2, is a four port of which arms 3 and 4 are connected by a section of waveguide or transmission line forming the annular ring. Arms 1 and 2 form the waveguide to which the ring guide is coupled.

The scattering properties of the junction will be described by the reflection coefficients ρ_{nn}, transmission coefficients τ_{nm}, and by the coupling factors k_r and k_h for the waves passing from one waveguide into the other. The simplifying assumption is made that port 2 is terminated with a matched termination and that the circuit is fed by port 1. The general case, which includes a reflected wave h_2 incident into port 2, can be considered as a superposition of the waves in the two cases when the circuit is fed alternately by port 1 and 2.

The reflected wave r and the incident wave h in the region of the junction are related by a scattering matrix S:

$$r = Sh. \tag{1}$$

Because of symmetry of the junction, reciprocity, and matched termination, the scattering matrix has the form

$$S = \begin{pmatrix} \rho_0 & \tau & k_h & k_r \\ \tau & \rho_0 & k_r & k_h \\ k_h & k_r & \rho_0 & \tau \\ k_r & k_h & \tau & \rho_0 \end{pmatrix}, \tag{2}$$

where k_h and k_r are the coupling factors between the

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† Dept. of Elec. Eng., Ohio State University, Columbus, Ohio.
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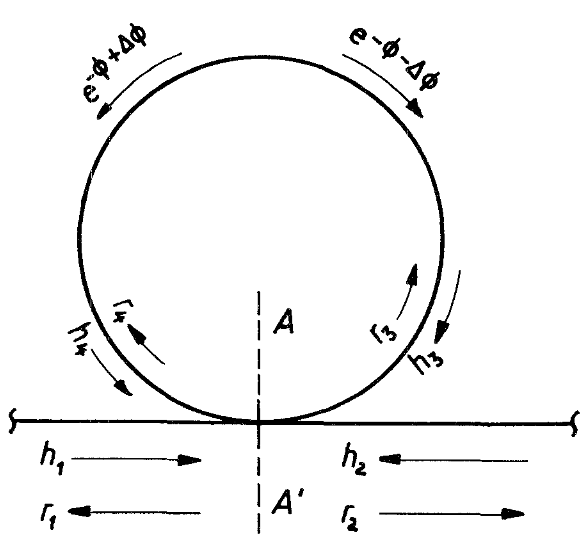


Fig. 1—Nonreciprocal ring circuit.

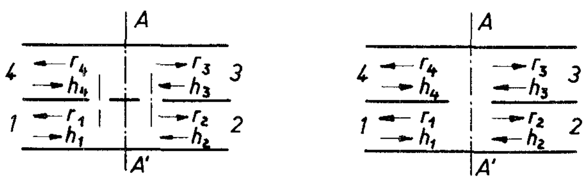


Fig. 2—Coupling conditions: (a) directional coupling, (b) nondirectional coupling.

coupled waveguides in the forward and reverse directions;

$$k_h = \tau_{13} = \tau_{31} = \tau_{24} = \tau_{42}, \tag{3}$$

$$k_r = \tau_{12} = \tau_{21} = \tau_{34} = \tau_{43}. \tag{4}$$

The scattering matrix yields a system of equations from which the different waves can be determined. The waves progressing in the ring guide in both directions to the junction are denoted by h_3 and h_4. With the magnitudes of h_3 and h_4 known, the total input reflection coefficient ρ_{in} and transmission coefficient in the main guide τ_{12} can be calculated,

$$\rho_{in} = \frac{r_1}{h_1} = \rho_0 + \frac{h_3 k_h + h_4 k_r}{h_1} \tag{5}$$

and

$$\tau_{12} = \frac{r_2}{h_1} = \tau + \frac{h_3 k_r + h_4 k_h}{h_1}\cdot \tag{6}$$

The properties of the ring circuit as a filter are completely defined by the waves in the ring h_3 and h_4. The superposition of h_3 and h_4 yields ρ_{in} and τ_{12} which describe the reaction of the ring circuit on the main guide. A similar relation can be obtained for the case in which energy is coupled out of the ring guide by a second coupling element into a second waveguide. Under this condition, the circuit operates as a transmission type filter. Also in this case h_3 and h_4 define completely the properties of the circuit. Therefore, the investigation will concentrate on the ringwaves h_3 and h_4.

Coupling Conditions

It is possible to distinguish between three distinct coupling conditions: 1) directional ($q=0$), 2) nondirectional ($q=1$), and 3) semidirectional ($0<q<1$) coupling, where $q=k_r/k_h$. The coupling depends on the properties of the coupling element.

The results of an investigation of the coupling problem are shown in Appendix I. Capacitive coupling between the waveguides is considered. According to the derivations, the junction data can be described in a general manner by approximate expressions for ρ_0 and τ;

$$\rho_0 \approx -iq\,|k_h| \approx -k_r \tag{7}$$

and

$$\tau \approx 1 - |k_h|^2(1+q^2) - i\,|k_h|\,. \tag{8}$$

Eqs. (7) and (8) will permit a better interpretation of the results.

Description of Nonreciprocal Ring Waves by Poles and Zeros

The representation of ring waves h_4 and h_3 by poles and zeros is used to show the influence of nonreciprocity on the properties of the ring circuit. The equations are similar to those of Appendix II for the reciprocal case.

The wave propagation within the ring guide is characterized by (9) and (10)

$$h_3 = r_4 e^{-\phi} e^{-\Delta\phi}, \tag{9}$$

$$h_4 = r_3 e^{-\phi} e^{+\Delta\phi}, \tag{10}$$

where $e^{\Delta\phi}$ takes into account the nonreciprocity in both directions (Fig. 1). The introduction of (9) and (10) into the system of equations derived from (2), yields

$$r_3 = h_1 k_h \frac{1 - \tau e^{-\phi}e^{-\Delta\phi} + q\rho_0 e^{-\phi}e^{-\Delta\phi}}{(1-\tau e^{-\phi}e^{-\Delta\phi})(1-\tau e^{-\phi}e^{+\Delta\phi}) - \rho_0^2 e^{-2\phi}} \tag{11}$$

and

$$r_4 = h_1 k_h q \frac{1 - \tau e^{-\phi}e^{+\Delta\phi} + \dfrac{1}{q}\rho_0 e^{-\phi}e^{+\Delta\phi}}{(1-\tau e^{-\phi}e^{-\Delta\phi})(1-\tau e^{-\phi}e^{+\Delta\phi}) - \rho_0^2 e^{-2\phi}}\cdot \tag{12}$$

It follows that

$$h_4 = h_1 k_h e^{+\Delta\phi} \frac{e^{\phi} - e^{\phi_3}}{(e^{\phi} - e^{\phi_1})(e^{\phi} - e^{\phi_2})}, \tag{13}$$

$$h_3 = h_1 k_h q e^{-\Delta\phi} \frac{e^{\phi} - e^{\phi_4}}{(e^{\phi} - e^{\phi_1})(e^{\phi} - e^{\phi_2})}, \tag{14}$$

in which

$$e^{\phi_{1,2}} = \tau \cosh \Delta\phi \pm \rho_0 \sqrt{\frac{\tau^2}{\rho_0^2}(\sinh^2 \Delta\phi + 1)}, \tag{15}$$

$$e^{\phi_3} = e^{-\Delta\phi}(\tau - q\rho_0), \tag{16}$$

$$e^{\phi_4} = e^{+\Delta\phi}\left(\tau - \frac{1}{q}\rho_0\right). \tag{17}$$

The poles $e^{\phi_{1,2}}$ are obtained as solutions of quadratic a equation when the denominators of (11) and (12), which are equal, are multiplied by $e^{2\phi}$ and equated to zero. Expressions for e^{ϕ_3} and e^{ϕ_4} are derived from the numerators.

Difference of Wave Velocity in Both Directions

As a significant case of nonreciprocity, the value of $\Delta\phi$ is assumed purely imaginary:

$$\Delta\phi = i\Delta\beta L.$$

This case results from differences of the wave velocities in both directions in the ring guide. Low dc magnetization of ferrites yields this condition.

If the variation $\Delta\beta L$ of the total phase shift due to nonreciprocity is small, the trigonometric functions derived from (15) may be replaced by the corresponding series and higher order terms may be neglected. Introduction of the approximate coupling data [(17) and (18)] gives

$$e^{\phi_{1,2}} \approx 1 - |k_h|^2(1 + q^2 + \tfrac{1}{2}m^2) - i|k_h|(1 \mp \sqrt{q^2 + m^2}), \quad (18)$$

$$e^{\phi_3} \approx 1 - |k_h|^2(1 + q^2 + \tfrac{1}{2}m^2) - i|k_h|(1 - q^2 + m), \quad (19)$$

and

$$e^{\phi_4} \approx 1 - |k_h|^2(1 + q^2 + \tfrac{1}{2}m^2) + i|k_h|m, \quad (20)$$

where $m = \Delta\beta L/|k_h|$ is the ratio of the phase shift difference to $|k_h|$ which is the coupling coefficient for the waves passing in the forward direction from the main guide into the ring guide. The coupling conditions (directional, nondirectional, and semidirectional) are defined by q.

According to (13) and (14), the poles e^{ϕ_1}, e^{ϕ_2} and the zeros e^{ϕ_3} and e^{ϕ_4} describe the frequency dependence of the ring waves h_4 and h_3, based on the same concept as used in Appendix II in connection with Fig. 7. The real parts $|k_h|^2(1+q^2+\frac{1}{2}m^2)$ show the influence of coupling and nonreciprocity on the Q value, and the imaginary parts of the poles show the influence on the resonant frequencies. Caution must be exercised if a zero coincides with a pole thus yielding a single-peak resonance curve.

The imaginary parts $Y_n = \text{Im}(e^{\phi_n})$ which show the influence of coupling and nonreciprocity on the resonant frequencies are plotted in Fig. 3 as a function of the coupling (q) for different values of $m = \Delta\beta L/|k_h|$. The magnitude of m is a measure of the nonreciprocity. The poles are shown in the diagram a, the zeros for the waves in both directions in diagram b. The diagrams show for which value of q and m the poles and zeros coincide, such as Y_1, Y_2, and Y_3 for $q=0$ and $m=0$ and Y_1 and Y_3 for $m=0$ and $q=1$. It is interesting to note that only in the case of purely directional coupling ($q=0$) the nonreciprocity does not dislocate coinciding poles and zeros. In all other cases, nonreciprocity leads to a splitting of poles and zeros. This is important in the case of nondirectional coupling ($q=1$) where the single-peak resonance curve changes into a double-peak curve due to nonreciprocity. Effects of this type are important in the case of the measurement of ferrite properties in a ring guide.

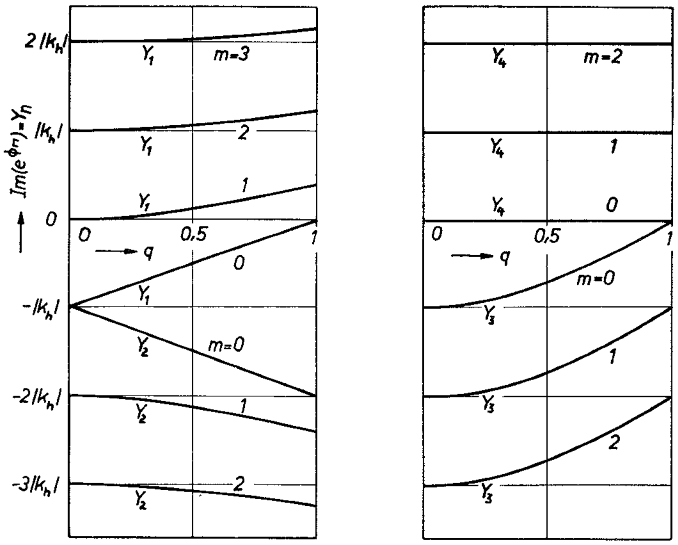


Fig. 3—Influence of nonreciprocity on the resonant frequencies. Imaginary parts of poles and zeros. ($q = k_r/k_h$; $m = \Delta\phi/|k_h|$.)

Excessive Attenuation in One Direction

Other interesting conditions result from excessive attenuation in one direction and negligible attenuation in the other direction in the ring guide. Under these conditions the ring waves are related by

$$h_3 = r_4 e^{-\phi} e^{-\Delta\phi} \quad (21)$$

and

$$h_4 = r_3 e^{-\phi}, \quad (22)$$

where

$$\Delta\phi = \Delta\alpha L. \quad (23)$$

The excessive attenuation in one direction can be produced by ferrite material magnetized near gyromagnetic resonance. An isolator as a section of the ring circuit has the same effect.

Calculation of the poles and zeros yields:

$$e^{\phi_{1,2}} \approx 1 - 2|k_h|^2 - i|k_h| - \frac{\Delta\phi}{2} \pm \sqrt{\frac{\Delta\phi^2}{4} - |k_h|^2}, \quad (24)$$

$$e^{\phi_3} \approx 1 - 2|k_h|^2 - \Delta\phi \quad (25)$$

and

$$e^{\phi_4} \approx 1 - 2|k_h|^2, \quad (26)$$

for nondirectional coupling which is the most interesting case ($k_r = k_h$). Fig. 4 shows the locus of the poles for varying excessive attenuation in one direction.

The quotient r_3/r_4 is another quantity which is of particular interest in the case of differing attenuation. It

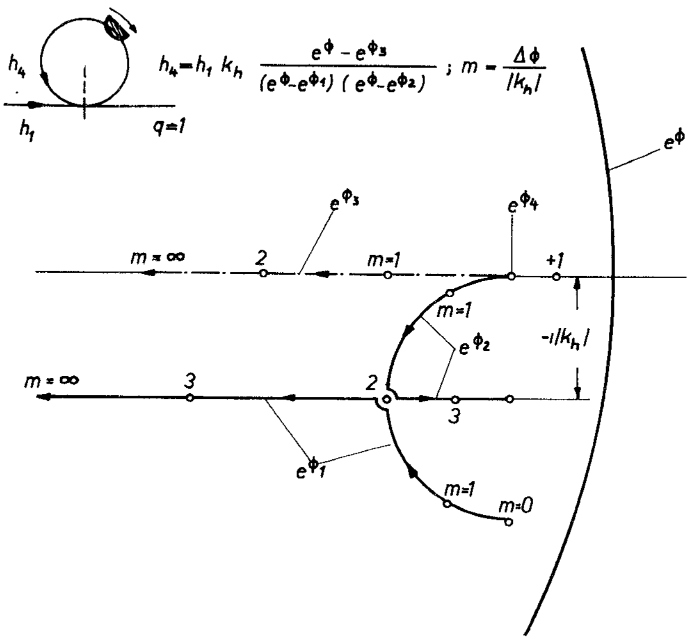


Fig. 4—Loci of poles and zeros of the ring waves for varying excessive attenuation. Nondirectional coupling.

may be shown from (11) and (12) that

$$\frac{r_3}{r_4} = \frac{e^{\phi} - e^{\phi_3}}{e^{\phi} - e^{\phi_4}}. \tag{27}$$

Eq. (27) can be treated in the same manner as one of the wave functions.

Additional Discontinuity in the Ring Guide

The ring guide has similar properties to those of a cavity with different wave modes which resonate at the same or at different frequencies. The latter case corresponds to a nonreciprocal ring guide. A single-peak resonance curve is obtained if no coupling exists between the waves in both directions in the ring guide. This occurs for purely directional coupling where the discontinuity of the coupling element is negligible ($\rho_0 = 0$). Consequently, the question of what happens if a discontinuity is introduced into a nonreciprocal, directionally coupled ring guide arises. This problem occurs in practice when a ferrite material probe is introduced into a ring circuit for determination of its material properties.

The ring waves are expressed by

$$h_3 = r_3\rho_1 e^{-\phi} + r_4(1 + \rho_1)e^{-\phi}e^{-\Delta i\beta L}, \tag{28}$$

$$h_4 = r_4\rho_1 e^{-\phi} + r_3(1 + \rho_1)e^{-\phi}e^{+\Delta i\beta L}, \tag{29}$$

where reflections result from a symmetrical discontinuity located diametrically opposite to the coupling element in the ring guide. The reflection coefficient of the discontinuity is ρ_1, and for the nonreciprocity, a difference of total phase shift in both directions $2\Delta\beta L$ is introduced.

The ring waves are described by

$$r_3 = h_1 k_h \frac{e^{\phi} - e^{\phi_3}}{e^{-\phi}(e^{\phi} - e^{\phi_1})(e^{\phi} - e^{\phi_2})} \tag{30}$$

and

$$r_4 = h_1 k_h 9 \frac{e^{\phi} - e^{\phi_4}}{e^{-\phi}(e^{\phi} - e^{\phi_1})(e^{\phi} - e^{\phi_2})}, \tag{31}$$

where the poles and zeros are defined by

$$e^{\phi_{1,2}} \approx \tau\left[1 - i\frac{\Delta p}{2} - \frac{(\Delta\beta L)^2}{2} \pm i\sqrt{(\Delta\beta L)^2 + \frac{\Delta p^2}{4}}\right], \tag{32}$$

$$e^{\phi_3} \approx \tau\left(1 - i\frac{\Delta p}{2}\right)e^{-i\Delta\beta L} \tag{33}$$

and

$$e^{\phi} - e^{\phi_4} \approx -i\tau\frac{\Delta p}{2}e^{-\phi}, \tag{34}$$

for small values of the discontinuity ($\rho_1 = -i\Delta p/2$) and of the nonreciprocity.

Evaluation of (30) through (34) shows that the ring waves in the reversed direction result directly from the discontinuity, (34), and that the discontinuity splits a coinciding pole and zero. Instead of a single-peak resonance curve without discontinuity, a double-peak curve is obtained.

Conclusion

It is desirable to treat the ring circuit under a variety of conditions which take into account coupling to an input and output waveguide. The representation of the waves in the ring guide by the poles and zeros of a rational function gives a satisfactory description of the ring circuit properties and shows clearly the influence of nonreciprocity under the different coupling conditions. Consideration of practical coupling conditions and the introduction of the corresponding relations (Appendix I) further improves the representation. The poles and zero representation permits calculation of the resonant frequencies, a simple graphical determination of resonance curves and a calculation of the loaded and unloaded Q values.

With respect to the nonreciprocity, two particular cases have been treated separately. One is characterized by a difference of wave velocity in both directions, the other by excessive attenuation in one direction. Both can be superimposed to show the influence of nonreciprocity in the general case.

The results obtained for a difference of the wave velocity show clearly the influence of the nonreciprocity on the resonance frequencies (Fig. 3) for the different coupling conditions described by $q = k_r/k_h$. Presence of an additional discontinuity in the ring guide in the case of directional coupling ($q = 0$) adds one more interesting operational condition. The additional discontinuity may result from a second coupling element by which energy is coupled out from the ring guide.

The case of an additional discontinuity is of particular interest when it is desired to determine the properties of ferrite materials in a directionally coupled ring guide. It is assumed that for directional coupling only waves in one direction progress in the ring guide. However, since the ferrite material probe represents a discontinuity, reflected waves occur in the opposite direction. These reflected waves contribute to a variation of the resonant frequency. This variation must be taken into account if the material properties are to be determined from the shift of the resonance frequency by perturbation methods.

The influence of the additional discontinuity can be visualized clearly in the poles and zero presentation. The discontinuity results in a split of a coinciding pole and zero which changes the zero-peak resonance curve into a double-peaked curve. The split of the pole and the zero must be taken into account in the determination of the Q value. The discontinuity can be avoided if two thin cylindrical ferrite material probes are inserted $\lambda_g/4$ apart, if the probe has the form of a rectangular disk of a length $\lambda_g/2$ or, if the probe extends over the total circumference.

The nonreciprocal ring circuit has interesting properties which differ from those of other circuits. Besides the application for determination of the ferrite material properties, especially for high field strengths, one can anticipate applications in other fields of microwaves.

Appendix I

Coupling Relations

Two waveguides are coupled together by a nondirectional coupling element in the cross-sectional plane AA'. In Fig. 5(a), a capacity C is shown as the coupling element. Derivation of the junction data yields

$$\rho_0 = -\frac{i\Delta p}{1+2i\Delta p}, \tag{35}$$

$$\tau = -\frac{1+i\Delta p}{1-i\Delta p} \tag{36}$$

and

$$k_h = k_r = k = \frac{i\Delta p}{1+2i\Delta p},$$

where

$$\Delta p = \omega C Z_0/2.$$

If high order terms are neglected, approximate expressions are obtained;

$$\rho_0 \approx -i\,|k_h|, \tag{37}$$

$$\tau \approx 1 - 2\,|k_h|^2 - i\,|k_h|. \tag{38}$$

Eqs. (37) and (38) also are valid for inductive coupling where $k_r = -k_h$.

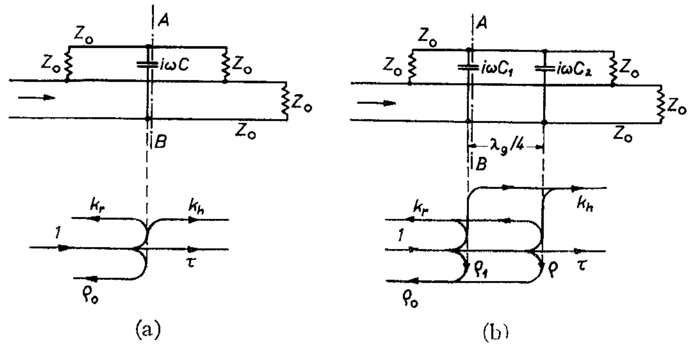


Fig. 5—Capacitive coupling. (Nondirectional and semidirectional coupling.)

The corresponding equivalent circuit for directional and semidirectional coupling is shown in Fig. 5(b). Coupling is performed by two capacitances $\lambda_g/4$ apart. Consequently,

$$\Delta p_1 = \omega C_1 Z_0/2, \qquad \rho_1 = -\frac{i\Delta p_1}{1+2i\Delta p_1},$$

$$\Delta p_2 = \omega C_2 Z_0/2, \qquad \rho_2 = -\frac{i\Delta p_2}{1+2i\Delta p_2}.$$

If all junction data are reduced with respect to the cross sectional plane AA', the following equations are obtained:

$$[\rho_0]_{AA^1} = \frac{\rho_1 - \rho_2 - 2\rho_1\rho_2}{1+\rho_1\rho_2},$$

$$[\tau]_{AA^1} = \frac{1+\rho_1+\rho_2+\rho_1\rho_2}{1+\rho_1\rho_2},$$

$$k_h + k_r = -2\rho_1,$$

$$k_h - k_r = -2\rho_2,$$

$$q = k_r/k_h.$$

The dropping of higher order terms and the disregarding of reflections between the coupling capacitances yields

$$\rho_0 \approx -iq\,|k_h| \approx -k_r \tag{39}$$

and

$$\tau \approx 1 - |k_h|^2(1+q^2) - i\,|k_h|. \tag{40}$$

Appendix II

Poles and Zeros of the Ring Waves

The properties of the ring circuit can be described by the waves h_4, h_3 progressing in the ring guide in both directions. These waves are characterized in the case of the reciprocal ring guide by

$$h_3 = r_4 e^{-\phi},$$

$$h_4 = r_3 e^{-\phi},$$

$$\phi = (\alpha + i\beta)L,$$

where L is the circumference and α and β are the attenuation and phase constants, respectively, of the ring guide. Combination with the system of equations corresponding to the matrix (2) gives

$$h_4 = h_1 e^{-\phi} \frac{k_h(1 - \tau e^{-\phi}) + k_r \rho_0 e^{-\phi}}{(1 - \tau e^{-\phi})^2 - \rho_0^2 e^{-2\phi}},$$

$$h_3 = h_1 e^{-\phi} \frac{k_r(1 - \tau e^{-\phi}) + k_h \rho_0 e^{-\phi}}{(1 - \tau e^{-\phi})^2 - \rho_0^2 e^{-2\phi}}.$$

Transformation and introduction of $q = k_r/k_h$ yields

$$h_4 = h_1 k_h \frac{e^{\phi} - (\tau - q\rho_0)}{[e^{\phi} - (\tau + \rho_0)][e^{\phi} - (\tau - \rho_0)]} = h_1 k_h \frac{e^{\phi} - e^{\phi_3}}{(e^{\phi} - e^{\phi_1})(e^{\phi} - e^{\phi_2})} \quad (41)$$

and

$$h_3 = h_1 k_h q \frac{e^{\phi} - (\tau - \rho_0/q)}{[e^{\phi} - (\tau + \rho_0)][e^{\phi} - (\tau - \rho_0)]} = h_1 k_h q \frac{e^{\phi} - e^{\phi_4}}{(e^{\phi} - e^{\phi_1})(e^{\phi} - e^{\phi_2})}. \quad (42)$$

Eqs. (41) and (42) describe the waves h_4 and h_3 by poles (e^{ϕ_1}, e^{ϕ_2}) and zeros (e^{ϕ_3}, e^{ϕ_4}) which correspond to the "resonances" and "antiresonances" in the language of network theory.

The representation by poles and zeros is advantageous because it aids the analysis of the frequency dependence of the wave functions. It permits simple graphical determination of $|h_4|$ and $|h_3|$ as a function of frequency.

The locus of e^{ϕ} as a function of frequency is a circle with the radius $e^{\alpha L}$ where $e^{-\alpha L}$ expresses the attenuation of the waves along the circumference of the ring guide. The attenuation is assumed to be constant in a small frequency range near resonance. The poles and zeros e^{ϕ_n} are complex quantities which also are approximately constant near resonance. Fig. 6 shows these magnitudes in the complex plane. The point T which corresponds to $e^{\phi} = e^{\alpha L} e^{i(2\pi/\lambda_g)L}$ travels with increasing frequency along the circle in the direction of the arrow. With P_1, P_2 for the poles and Z for the zero, the absolute value $|h_4|$ is proportional to $\overline{TZ}$ divided by the product of $\overline{TP}_1 \times \overline{TP}_2$. Plotting $|h_4|$ as a function of frequency yields the resonance curve. When T passes near a pole the resonance curve has a peak. Near a zero the resonance curve has a minimum.

Eqs. (41) and (42) show that the locations of the poles and zeros are dependent on ρ_0, τ, and q. Introduction of the approximate values derived in Appendix I yields

$$e^{\phi_1} = 1 - |k_h|^2(1 + q^2) - i|k_h|(1 + q),$$
$$e^{\phi_2} = 1 - |k_h|^2(1 + q^2) - i|k_h|(1 - q),$$
$$e^{\phi_3} = 1 - |k_h|^2(1 + q^2) - i|k_h|(1 - q^2)$$

and

$$e^{\phi_4} = 1 - |k_h|^2(1 + q^2).$$

If the poles and zeros are plotted in the complex plane as a function of q, the curves of Fig. 7(b) are obtained. The coupling is defined by q, where directional, semidirectional, and nondirectional coupling are signified by $q=0$, $0<q<1$, and $q=1$, respectively. For three values, $q=0$, 0.5, 1, the resonance curves are shown in Fig. 7(a).

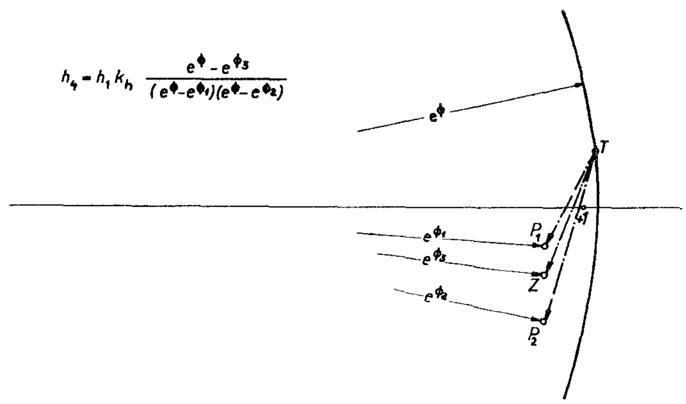
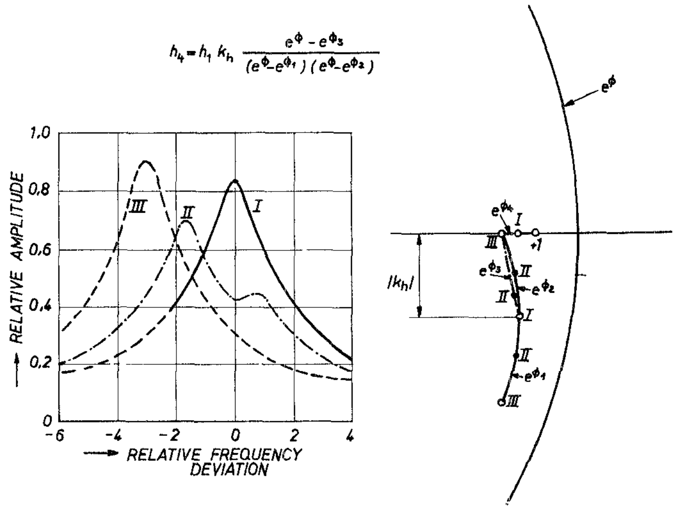


Fig. 6—Complex poles and zeros of the ring wave h_4.



(a) (b)

Fig. 7—Loci of poles and zeros for varying coupling and corresponding resonance curves.

For $q=0$ (I), the two poles and the zero coincide. Consequently, a single-peak resonance curve is obtained. The same result is obtained for nondirectional coupling ($q=1$), (III). For values $0<q<1$ the poles and the zeros have different locations (II) and a double peak is obtained as shown in Fig. 7(a).

The poles and zeros representation permits a simple approximate calculation of the Q value. The Q value may be derived for a circuit with a single-peak resonance curve from the half-power bandwith. The half-power bandwith corresponds to travel on the e^{ϕ} circle over an arc which equals twice the distance of the pole e^{ϕ_0} from the circle.

Use of this concept yields

$$Q = \frac{\beta L}{2(\alpha L + 1 - e^{\phi_0})} \left(\frac{\lambda g}{\lambda_0}\right)^2$$

for the loaded Q value, and

$$Q = \frac{\beta}{2\alpha} \left(\frac{\lambda g}{\lambda_0}\right)^2$$

for the unloaded value where the coupling approaches zero ($|k_h| \rightarrow 0$).